

Mark Scheme (Results)

Summer 2018

Pearson Edexcel International Advanced Level In Mathematics Mechanics M3 (WME03) Paper 01

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- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

General Marking Guidance

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PEARSON EDEXCEL IAL MATHEMATICS

General Instructions for Marking

- 1. The total number of marks for the paper is 75.
- 2. The Edexcel Mathematics mark schemes use the following types of marks:

'M' marks

These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.

e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.

The following criteria are usually applied to the equation.

To earn the M mark, the equation

- (i) should have the correct number of terms
- (ii) be dimensionally correct i.e. all the terms need to be dimensionally correct e.g. in a moments equation, every term must be a 'force x distance' term or 'mass x distance', if we allow them to cancel 'g' s.

For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned. e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

'A' marks

These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. MO A1 is impossible.

'B' marks

These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. - follow through - marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod benefit of doubt
- ft follow through
- the symbol √ will be used for correct ft
- cao correct answer only
- cso correct solution only. There must be no errors in this part of the question to obtain this mark
- isw ignore subsequent working
- awrt answers which round to
- SC: special case
- oe or equivalent (and appropriate)
- dep dependent
- indep independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- The second mark is dependent on gaining the first mark
- 4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.
- 5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.
- 6. If a candidate makes more than one attempt at any question:
 - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
 - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.
- 7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.
- Omission or extra g in a resolution is an accuracy error not method error.
- Omission of mass from a resolution is a method error.
- Omission of a length from a moments equation is a method error.
- Omission of units or incorrect units is not (usually) counted as an accuracy error.
- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.
- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.
- Use of g = 9.81 should be penalised once per (complete) question.

N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),.....then that working can only score marks for that part of the question.
- Accept column vectors in all cases.
- Misreads if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft
- Mechanics Abbreviations
 - M(A) Taking moments about A.
 - N2L Newton's Second Law (Equation of Motion)
 - NEL Newton's Experimental Law (Newton's Law of Impact)
 - HL Hooke's Law
 - SHM Simple harmonic motion
 - PCLM Principle of conservation of linear momentum
 - RHS, LHS Right hand side, left hand side.

June 2018 WME03 M3 Mark Scheme

Question Number	Scheme	Marks
1.	$T = \frac{29.4(0.5 - l)}{l}$	
	$1.5g = \frac{29.4(0.5 - l)}{l}$	M1A1
	$1.5 = \frac{1.5 - 3l}{l}$	
	3l = 1 $l = 0.33$ or 0.333 m must be 2 or 3 sf	dM1A1 [4]
M1 A1 dM1 A1	Attempt Hooke's Law with one unknown (natural length or extension but no Extension = 0.5 scores M0. T or 1.5 for this mark Fully correct vertical equation Solve their equation depends on the first M mark Correct natural length Ignore $l = \frac{1}{3}$ if 0.33 or 0.33 also seen	t both)
NB:	If extension is used as the unknown and work not completed to give the natural M1A1 only available.	ral length, first

Question Number	Scheme	Marks
2(a)	$\ddot{x} = -\omega^2 x$	
	$\omega^2 = \frac{0.5}{0.02}, \implies \omega = 5$ Period = $\frac{2\pi}{5}$ s Accept 1.3, 1.26 or better	M1,A1 M1A1 (4)
(b)	$v^{2} = \omega^{2} \left(a^{2} - x^{2} \right)$ $0.3^{2} = \frac{0.5}{0.02} \left(a^{2} - 0.02^{2} \right)$	M1A1ft
	$a = 0.06324$ Accept 0.063 or better $\left(\text{exact is } \frac{\sqrt{10}}{50} \text{ oe}\right)$	A1 (3)
(c)	$\frac{1}{2}a = a\sin\omega t$	
	$t = \frac{1}{\omega} \sin^{-1}\left(\frac{1}{2}\right), = \frac{\pi}{30}$ (= 0.104)	M1,A1
	Total time = $4 \times \frac{\pi}{30}$, = $\frac{2\pi}{15}$ s Accept 0.42, 0.419 or better	dM1,A1 (4)
(a)		
M1 A1	Use \ddot{x} or $a = -\omega^2 x$ with $\ddot{x} = 0.5$ and $x = 0.02$ to form an equation for ω^2 . A with or without a minus sign. Obtain correct value for ω or ω^2 from correct working ie no sign errors $(\omega^2 = -25, \omega = 5 \text{ scores M1A0})$	llow equation
M1	Use period $\frac{2\pi}{n}$ with their ω	
A1 (b)	Correct period, exact or decimal (can score M1A0M1A1 in (a))	
M1	Use $v^2 = \omega^2 (a^2 - x^2)$ with $v = 0.3, x = 0.02$ and their ω	
A1ft A1 (c)	Correct equation, follow through their ω Correct value of a , 2 sf or better (inc exact)	
M1	Use $x = a \sin \omega t$ or $x = a \cos \omega t$ with their ω and $x = \pm \frac{1}{2}a$. May use their value	alue for <i>a</i> or use
A1 dM1	a Obtain the correct time from the centre or end of oscillation (depends on the Correct method to complete to obtain the required time. Depends on the first If $x = a \cos \omega t$ is used this mark needs "period $-4 \times$ time found" (or any eq Correct total time, exact or decimal	equation used) M mark of (c).

Question Number	Scheme	Marks
3 (a)	$\cos A = \sin B = \frac{4}{5}, \sin A = \cos B = \frac{3}{5}$ $T_A \cos A = T_B \cos B + mg$	B1 any of these M1A1
	$T_{A} \sin A + T_{B} \sin B = mr\omega^{2}, = 4l \sin A \times m\omega^{2} \left(= \frac{12}{5} ml\omega^{2} \right)$	M1A1,A1
	$\begin{bmatrix} \frac{4}{5}T_A = \frac{3}{5}T_B + mg \\ \frac{3}{5}T_B + mg \end{bmatrix}$	
	$\frac{3}{5}T_A + \frac{4}{5}T_B = 4lm\omega^2 \times \frac{3}{5}$	
(i)	$T_A = \frac{m}{25} \left(36l\omega^2 + 20g \right) \qquad \text{oe}$	dM1 (either) A1
(ii)	$T_B = \frac{m}{25} \left(48l\omega^2 - 15g \right) \qquad \text{oe}$	A1 (9)
(b)	$T_B \geqslant 0 \Rightarrow 48l\omega^2 \geqslant 15g$	M1
	$\omega \geqslant \sqrt{\frac{15g}{48l}} = \frac{1}{4}\sqrt{\frac{5g}{l}}$	
	$R \leqslant 2\pi \times 4\sqrt{\frac{l}{5g}}$	dM1
	(least) $k = 8$	A1 cso (3) [12]
(a)B1 M1	Any correct sine or cosine seen explicitly or used. Resolve vertically	
A1	Correct equation, can have trig functions (as shown) or numerical values for	these
M1 A1	Equation of motion along the radius, acceleration in either form, r for radius Left hand side correct, trig functions or numerical values accepted	accepted here
A1	Correct right hand side. Acceleration to be $r\omega^2$ with radius $4l \sin A$ or $\frac{12}{5}$	
(i)dM1	Solve their 2 equations to obtain an expression for either tension Depends on both previous M marks	
A1 (ii)A1 NB	Correct expression for T_A , any equivalent form Correct expression for T_B , any equivalent form Ignore (i) (ii) labels provided it is clear which is the tension in AC and which in BC	is the tension
(b) M1	Use $T_B \geqslant 0$ to obtain an inequality for ω^2 . Must use \geqslant	
dM1	Use $R = \frac{2\pi}{\omega}$ with their inequality for ω^2 to obtain an inequality for R . Allow use of T	
A1cso	instead of R . Deduce the correct value for k . The value must be shown explicitly but need "least". (no inequality allowed)	not include

Question Number	Scheme	Marks
4	$0.5u = 1.5$ $u = 3 \text{ m s}^{-1}$	B1
	Work done against friction = $0.7 \times 0.5 \cos 30g \times 0.6$	M1A1
	Initial EPE = $\frac{\lambda \times 0.6^2}{2 \times 0.6} \left(= \frac{0.6\lambda}{2} = 0.3\lambda \right)$	B1
	$\frac{\lambda \times 0.6^2}{2 \times 0.6} + \frac{1}{2} \times 0.5 \times 9 = 0.7 \times 0.5 \cos 30g \times 0.6 + 0.5 \times g \times 0.6 \sin 30$	M1A1A1 Ft EPE and Work
	$\lambda = 3.340 = 3.3 \text{ or } 3.34$	A1 [8]
B1 M1 A1 B1 M1 A1ft A1ft A1	Correct value for u , seen explicitly or used. Attempt the work done against friction. Weight must be resolved (sin/cos interchange accepted.) Distance moved to be 0.6 m. Mass can be 0.5 or m Correct work done. Mass can be 0.5 or m Allow both of the above marks if the work done against friction is embedded in some incorrect work eg including other forces to form a resultant force. Correct initial EPE Need not be simplified. The work done and the EPE may not be shown explicitly. Check the equation if necessary. Attempt a complete work-energy equation. Must have an EPE, a GPE, a KE and a (dimensionally correct) work against friction term. The final KE may be included provided it becomes 0 here or later. EPE term must be of the form $\frac{k\lambda x^2}{l}$ $k = \frac{1}{2}$, 1 or 2 Deduct one per error. Follow through their EPE and work. Correct value of λ , 2 or 3 sf only.	

Question Number	Scheme	Marks
5(a)	$0.8v \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{4}{\left(x+1\right)^3}$	M1A1
	$v\frac{\mathrm{d}v}{\mathrm{d}x} = \frac{5}{\left(x+1\right)^3}$	
	$\frac{1}{2}v^2 = -\frac{5}{2(x+1)^2} (+c)$ $(t=0, v=0, x=0) \Rightarrow c = \frac{5}{2}$	dM1A1
	$(t=0, v=0, x=0) \implies c = \frac{5}{2}$	ddM1
	$v^{2} = 5\left(1 - \frac{1}{(x+1)^{2}}\right) = 5\left(\frac{(x+1)^{2} - 1}{(x+1)^{2}}\right)$	A1cso (6)
(b)	$v = \frac{\mathrm{d}x}{\mathrm{d}t} = \sqrt{5\left(\frac{(x+1)^2 - 1}{(x+1)^2}\right)}$ $\int \frac{(x+1)}{\sqrt{(x+1)^2 - 1}} \mathrm{d}x = \int \sqrt{5} \mathrm{d}t$	M1
	$\int \frac{(x+1)}{\sqrt{(x+1)^2 - 1}} dx = \int \sqrt{5} dt$	M1A1
	$((x+1)^2 - 1)^{\frac{1}{2}} = \sqrt{5}t \ (+k)$	M1A1
	$t = 0, x = 0 \implies k = 0$	
	$(x+1)^2 - 1 = 5t^2$ $x = \sqrt{5t^2 + 1} - 1$ Dist $= \sqrt{81} - 1 - (\sqrt{21} - 1) = 9 - \sqrt{21}$ or 4.417 m (accept 4.4 or better)	
	Dist = $\sqrt{81} - 1 - (\sqrt{21} - 1) = 9 - \sqrt{21}$ or 4.417 m (accept 4.4 or better)	M1A1cso(7) [13]
(a)M1	Attempt an equation of motion with acceleration = $v \frac{dv}{dx}$. Can be implied by	subsequent
A1	work. Fully correct equation. Can be implied by subsequent work.	
dM1	Attempt the necessary integration. $\frac{k}{(x+1)^3} \to \pm \frac{k'}{(x+1)^2}$ Depends on the first	t M mark
A1 ddM1	Correct integration (not ft). Constant may be omitted. Use the given initial conditions to obtain a value for the constant. Depends on both	
A1cso	preceding M marks. Initial conditions need not be shown explicitly. Reach the given answer in the form shown in the question. No errors in the v	work.

Question Number	Scheme	Marks
(b)M1 M1 A1 M1 A1 M1 A1 A1 A1 M1	Replace v with dx/dt in either the answer in (a) (with or without taking square root first) or in an equivalent expression drawn from their working in (a) Separate the variables ready to integrate. Must reach an integrand which can be integrated. All correct so far Attempt the integration by any valid means (eg inspection or substitution). Must include $\frac{(x+1)}{\sqrt{(x+1)^2-1}} \rightarrow A\left((x+1)^2-1\right)^{\frac{1}{2}} \text{ or } \frac{(x+1)}{\sqrt{x^2+2x}} \rightarrow B\left(x^2+2x\right)^{\frac{1}{2}} \text{ oe}$ Correct integration (not ft). Constant may be omitted. Use the given values of t to obtain the distances OA and OB and hence the distance AB . The function must have been integrated but third M mark may have been lost. Correct distance. Exact or decimal accepted. Constant must have been included and shown to be 0.	
(a)	By definite integration: $0.8v \frac{dv}{dx} = \frac{4}{(x+1)^3}$ $\int_0^v v dv = \int_0^x \frac{5}{(x+1)^3} dx$	M1A1
	$\left[\frac{1}{2}v^2\right]_0^v = \left[-\frac{5}{2(x+1)^2}\right]_0^x \qquad \text{(Integration condition as main scheme)}$ $\frac{1}{2}v^2 = -\frac{5}{2(x+1)^2} + \frac{5}{2}$ $v^2 = 5\left(1 - \frac{1}{(x+1)^2}\right) = 5\left(\frac{(x+1)^2 - 1}{(x+1)^2}\right)$	dM1A1 Ignore limits ddM1 Sub correct limits A1cso (6)
(b)	Complete solution following definite integration requires distances O to A and O to B being found followed by finding their difference. $v = \frac{dx}{dt} = \sqrt{5\left(\frac{(x+1)^2 - 1}{(x+1)^2}\right)}$ $\int_0^x \frac{(x+1)}{\sqrt{(x+1)^2 - 1}} dx = \int_0^T \sqrt{5} dt$	M1A1 ignore limits
	$\left[\left((x+1)^2 - 1 \right)^{\frac{1}{2}} \right]_0^X = \left[\sqrt{5}t \right]_0^T$ $(X+1)^2 - 1 = 5T^2 \qquad X = \sqrt{5T^2 + 1} - 1$ $\text{Dist} = \sqrt{81} - 1 - \left(\sqrt{21} - 1 \right) = 9 - \sqrt{21} \text{or 4.417 m} \text{(accept 4.4 or better)}$	Integration only. Ignore limits M1A1 Correct completion

Question Number	Scheme	Marks	
6(a)	$(\pi) \int y^2 x dx = (\pi) \int_0^h \left(\frac{r}{h}\right)^2 x^3 dx \text{OR} \left(\pi\right) \int y^2 x dx = (\pi) \int_0^h \left(r - \frac{r}{h}x\right)^2 x dx$	M1	
	$= \left(\pi\right) \left(\frac{r}{h}\right)^{2} \left[\frac{x^{4}}{4}\right]_{0}^{h}, = \left(\pi\right) \frac{r^{2}h^{4}}{4h^{2}} \left(=\left(\pi\right) \frac{r^{2}h^{2}}{4}\right)$	dM1,A1	
	$\overline{x} = (\pi) \frac{r^2 h^2}{4} \div \frac{1}{3} (\pi) r^2 h = \frac{3}{4} h$	M1A1cso (5)	
(b)	cone hemisphere S		
(2)	Mass $5m km m(5+k)$		
	Distance from $O = \frac{1}{4} \times 6r = (-)\frac{3}{8}r = \overline{x}$	B1	
	$5m \times \frac{3}{2}r - km \times \frac{3}{8}r = m(5+k)\overline{x}$	M1A1ft	
	$\overline{x} = \frac{3r(20-k)}{8(5+k)}$	A1 (4)	
(c)	Angle between AO and vertical = 30° or angle between axis and vertical = 60°	B1	
	$\tan 30 = \frac{\overline{x}}{r}, = \frac{3(20-k)}{8(5+k)}$ OR $\tan 60 = \frac{r}{\overline{x}}, = \frac{8(5+k)}{3(20-k)}$	M1,A1ft	
	k = 4.844 = 5	A1cso (4) [13]	
(a)	$II_{-2} \left(-\right) \int_{-2}^{2} \frac{1}{2} dx $	1	
M1	Use $(\pi) \int y^2 x dx$ to obtain an integral in x. Integral should be one of the form	is snown. π and	
dM1	limits need not be shown Attempt the integration; π and limits need not be shown. Depends on the pr	revious M mark.	
A1	Substitute (correct) limits to obtain correct result. π not needed, no need to s	simplify.	
	Second integral gives $\frac{r^2h^2}{12}$ or $\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right)r^2h^2$ on substitution of limits		
M1	Divide their result by the volume (given formula or an integral), π to be included in numerator and denominator or in neither. Can have the distance from the vertex or the distance from the base. Must reach $\bar{x} =$		
A1cso	Correct given result; no errors in the working.		
	If the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen to justify the distance from the base has been found $h - \frac{1}{4}h$ must be seen for	the answer.	
	Special Case: Cone base radius r and height $6r$		
	Equation is $y = \frac{1}{6}x$ Integral is $(\pi) \int_0^{6r} \frac{x^3}{36} dx$ or $(\pi) \int_0^{6r} \left(r - \frac{x}{6}\right)^2 x dx$		
	Award M marks only (if earned)		

Question Number	Scheme	Marks
(b)		
(b) B1	Correct distances shown explicitly or used. Negative sign may be missing he	ra
M1	Moments equation including a minus sign. Must use the given masses (or ratio of them), not volumes.	
A1ft	Fully correct equation follow through their distances	
A1	Correct expression for the required distance. Must be positive but can include modulus sign. $(k-20)$ in the numerator without modulus signs scores A0.	
	Equivalents accepted but not fractions within fractions.	
(c)		
B1	For either of the angles, shown explicitly or used. May be seen on a diagram	
M1	$\tan 30 \text{ or } 60 = \frac{\overline{x}}{r} \text{ or } \frac{r}{\overline{x}}$ (ie 30 or 60 and fraction either way up)	
A1ft	Fully correct equation. Follow through their \bar{x}	
A1cso	k = 5	

Question Number	Scheme	Marks
7 (a)	$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mgr(1-\cos\alpha)$	M1A1A1
	$mg\cos\alpha \ (-R) = m\frac{v^2}{r}$	M1A1
	$gr\cos\alpha = u^2 + 2gr(1-\cos\alpha)$	dM1
	$\cos \alpha = \frac{1}{3gr} \left(u^2 + 2gr \right)$	A1 cso (7)
(b)	At top $mg - R_{top} = \frac{mu^2}{r}$	
	For circular motion, $R_{top} > 0$: $\frac{u^2}{r} < g$, $u < \sqrt{gr}$	M1,A1cso (2)
ALT:	$\cos \alpha < 1 \qquad u^2 + 2gr < 3gr, u < \sqrt{gr}$	M1,A1cso (2)
(c)	$\frac{1}{2}m \times \frac{9}{2}gr - \frac{1}{2}mu^2 = 2mgr$ $u^2 = \frac{1}{2}gr$	M1A1 A1
	$\cos \alpha = \frac{1}{3gr} \left(\frac{1}{2} gr + 2gr \right)$	
	$\cos \alpha = \frac{5}{6}$	dM1A1cao (5) [14]
(a) M1 A1A1	Energy equation with a difference of KE terms and one PE term or a difference m in all terms or none but must be clear it is an energy equation and not $v^2 = -1$ each error	·
M1	Attempt an equation of motion along the radius at A. R need not be included. Weight must be resolved, acceleration in either form. Accept $v^2 = rg \cos \alpha$	
A1 dM1	Correct equation with or without R . Acceleration in form shown Eliminate v between the two equations (and set $R = 0$ if R was included) Depends on both previous M marks.	
A1cso	Correct given result (in the form shown) with no errors in the working. At least one intermediate step between $gr \cos \alpha = u^2 + 2gr(1-\cos \alpha)$ and the answer must be shown.	
(b)		1
M1	Attempt an equation of motion along the radius at the top and use $R_{top} > 0$ to inequality for u^2 Must use >	obtain an
A1cso ALT	Correct given result M1 Use $\cos \alpha < 1$ in the result from (a) to obtain an inequality for u^2 A1cso	As above

Question Number	Scheme	Marks	
(c) M1	Energy equation from top to reaching the plane. Difference of KE terms and needed <i>m</i> in all terms or none (see (a))	a PE term	
A1 A1 dM1 A1cao	-1 each error Use their expression for u^2 in the result given in (a) to obtain a numerical value Complete to 5/6. Accept 0.83 or better.	ue for $\cos \alpha$	
ALT 1	Alternatives for (c) Use energy from A to the plane		
M1 A1A1	$\frac{1}{2}m \times \frac{9}{2}gr - \frac{1}{2}mv^2 = mgr(1 + \cos\alpha)$ M1 equation with correct no of terms and dimensionally correct A1A1 -1 each error		
M1 A1cao	Use their equation of motion from (a) to find an expression for v^2 and complete to a numerical value for $\cos \alpha$ Complete to 5/6. Accept 0.83 or better.		
ALT 2	Using SUVAT equations: Let speed at A be v_A where $v_A^2 = u^2 + 2gr(1 - \cos \alpha)$ (from the energy equation in (a)) Horiz comp = $v_A \cos \alpha$ Vert comp = $v_A \sin \alpha$ Vert comp at the plane = V where $V^2 = (v_A \sin \alpha)^2 + 2gr(1 + \cos \alpha)$ (speed at plane) ² = $V^2 + (v_A \cos \alpha)^2$ = $(v_A \sin \alpha)^2 + 2gr(1 + \cos \alpha) + (v_A \cos \alpha)^2$ $\frac{9gr}{2} = v_A^2 + 2gr(1 + \cos \alpha)$ $\frac{9gr}{2} = u^2 + 2gr(1 - \cos \alpha) + 2gr(1 + \cos \alpha)$ $\frac{9gr}{2} = u^2 + 4gr \Rightarrow u^2 = \frac{1}{2}gr$ $\cos \alpha = \frac{1}{3gr}(\frac{1}{2}gr + 2gr) \Rightarrow \cos \alpha = \frac{5}{6}$	M1A1A1 M1A1cao	
M1 A1A1 M1 A1cao	Find the speed at A , obtain horizontal and vertical components of this speed. Obtain the vertical component of the speed at the plane using SUVAT Obtain the resultant speed at the plane, equate to $3\sqrt{\frac{gr}{2}}$ and eliminate v_A . For the equation obtained following the steps described above -1 each error in this equation. Equation need not be simplified. Deduct one mark if the horizontal and vertical components at A have been into As main scheme	terchanged	

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